

Math 116 Section 04

Midterm 4

Name _____

August 17, 2005

Instructor: Charles Cuell

Student Number _____

All solutions are to be presented on the paper in the space provided.
The exam is closed book, no calculators. Time for the exam is 50
minutes.

$$\begin{aligned}(1) \text{ (5 marks)} \text{ Evaluate } & \int_3^\infty e^{-x} dx \\&= \lim_{t \rightarrow \infty} \int_3^t e^{-x} dx \\&= \lim_{t \rightarrow \infty} -e^{-x} \Big|_3^t \\&= -\lim_{t \rightarrow \infty} (e^{-t} - e^{-3}) \\&= -(0 - e^{-3}) \\&= e^{-3}\end{aligned}$$

(2) (5 marks) Prove $\lim_{x \rightarrow 2} (x + 1) = 3$.

Let $\epsilon > 0$ and find δ :

$$\begin{aligned}|(x + 1) - 3| &< \epsilon \\ |x - 2| &< \epsilon\end{aligned}$$

So, let $\delta = \epsilon$. Then, the proof is:

$$\begin{aligned}|x - 2| &< \delta = \epsilon \\ |(x + 1) - 3| &< \epsilon\end{aligned}$$

(3) (5 marks) Evaluate $\lim_{x \rightarrow 0} x^x$.

Let $y = x^x$. Then $\ln y = x \ln x$.

$$\begin{aligned}\lim_{x \rightarrow 0} x \ln x &=^{0 \cdot -\infty} \lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{x}} \\ &=^{-\infty} \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}} \\ &= \lim_{x \rightarrow 0} -x \\ &= 0\end{aligned}$$

So, $\lim_{x \rightarrow 0} y = e^0 = 1$.

(4) (5 marks) Solve the initial value problem $\frac{dy}{dx} = x$, $y(0) = 1$

$$\frac{dy}{dx} = x$$
$$dy = xdx$$

$$\int dy = \int xdx$$
$$y = \frac{x^2}{2} + C$$

Using $y(0) = 1$, we get $1 = 0 + C$, so that the solution is $y = \frac{x^2}{2} + 1$.